

# Delayed birth of distillable entanglement in the evolution of bound entangled states

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The dynamical creation of entanglement between three-level atoms coupled to the common vacuum is investigated. For the class of bound entangled initial states we show that the dynamics of closely separated atoms generates stationary distillable entanglement of asymptotic states. We also find that the effect of delayed sudden birth of distillable entanglement occurs in the case of atoms separated by a distance comparable with the radiation wavelength.

PACS numbers: 03.67.Mn; 03.65.Yz; 42.50.-p

Keywords: three-level atoms; entanglement production; distillable entanglement

## I. INTRODUCTION

Dynamical creation of entanglement by the indirect interaction between otherwise decoupled systems has been studied by many researchers mainly in the case of two-level atoms interacting with the common vacuum. When the two atoms are separated by a distance small compared to the radiation wave length  $\lambda$ , there is a substantial probability that a photon emitted by one atom will be absorbed by the other and the resulting process of photon exchange produces correlations between the atoms. Such correlations may cause that initially separable states become entangled (see e.g. [1–4]). The case of three-level atoms is very interesting for many reasons. First of all, in a system of coupled multi - level atoms having closely lying energy states and interacting with the vacuum, quantum interference between different radiative transitions can occur, resulting in coherences in a system which are known as *vacuum - induced coherences*. Beside the usual effects such as collective damping and dipole-dipole interaction involving non-orthogonal transition dipole moments [5, 6], radiative coupling can produce here a new interference effect in the spontaneous emission. This effect manifests by the cross coupling between radiative transitions with *orthogonal* dipole moments [7] and is strongly dependent on the relative orientation of the atoms [8, 9]. All such collective properties of the system influence the quantum dynamics, which can significantly differ from a corresponding single atom dynamics. On the other hand, the theory of entanglement between the pairs of such atoms is much more complex than in the case of qubits. As is well known, there is no simple necessary and sufficient condition of entanglement, since the Peres-Horodecki separability criterion [10, 11] only shows that the states which are not positive after partial transposition (NPPT states) are entangled. But there can exist entangled states which are pos-

itive after this operation and such states are not distillable [12]. Hence all entangled states can be divided into two classes, one contains free entangled states that can be distilled using local operations and classical communication (LOCC) and the other consist of bound entangled states for which no LOCC strategy can be used to extract pure state entanglement. Since many effects in quantum information exploit pure maximally entangled states, only distillable states can be directly used for quantum communication. Contrary, nondistillable states involve some kind of irreversibility: we need pure entanglement to create them, but no pure entanglement can be obtained back from them [13].

The dynamics of entanglement between three-level atoms with vacuum-induced coherences was previously investigated mainly in the context of creation or degradation of NPPT states [14, 15]. In particular it was shown that for small distance between the atoms the system decays to a stationary state which can be entangled, even if the initial state was separable [14]. On the other hand, if the distance is comparable to the radiation wavelength, the dynamics brings all initial states into the asymptotic state in which both atoms are in their ground states but still there can be some transient entanglement between the atoms [15].

In the present paper, we investigate the dynamics of bound entangled initial states. There are some results concerning decoherence and disentanglement of bound entangled states [16], but here we focus on the process of dynamical creation of distillable entanglement due to the collective damping and cross coupling between the three-level atoms. For the specific bound entangled initial state and small interatomic distance we show that the asymptotic state is both entangled and distillable. Thus we obtain a stationary free entanglement. The same result is also valid for other initial states including separable states. So this dynamics of three - level atoms distinguishes distillable states, since all nontrivial asymptotic entangled states are also distillable. For larger distances, the dynamics of the bound entangled initial state is very peculiar: the system very quickly disentangle and only after some finite

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time there suddenly appears a distillable entanglement. (The similar phenomenon of delayed sudden birth of entanglement was observed in the case of two-level atoms [17]). So also in this situation the physical process of spontaneous emission can create some transient distillable entanglement out of the initially prepared bound entanglement.

## II. MIXED - STATE ENTANGLEMENT AND DISTILLATION

### A. Distillability of entanglement

Distillability of mixed entangled state  $\rho$  is the property that enables to convert  $n$  copies of  $\rho$  into less number of  $k$  copies of maximally entangled pure state by means of LOCC [18]. It is known that all pure entangled states can be reversibly distilled [19] and any mixed two-qubit entangled state is also distillable [20]. In general case, the following necessary and sufficient condition for entanglement distillation was shown in Ref. [12]: the state  $\rho$  is distillable if and only if there exists  $n$  such that  $\rho$  is  $n$  - copy distillable i.e.  $\rho^{\otimes n}$  can be filtered to a two-qubit entangled state. This condition is however hard to apply, since conclusions based on a few copies may be misleading [21]. More practical but not necessary condition is based on the reduction criterion of separability [22]. The criterion can be stated as follows: if a bipartite state  $\rho$  of a compound system  $AB$  is separable, then

$$\rho_A \otimes \mathbb{1} - \rho \geq 0 \quad \text{and} \quad \mathbb{1} \otimes \rho_B - \rho \geq 0 \quad (\text{II.1})$$

where

$$\rho_A = \text{tr}_B \rho, \quad \rho_B = \text{tr}_A \rho$$

As was shown in Ref.[23], any state that violates (II.1) is distillable, so if

$$\rho_A \otimes \mathbb{1} - \rho \not\geq 0 \quad \text{or} \quad \mathbb{1} \otimes \rho_B - \rho \not\geq 0 \quad (\text{II.2})$$

the state  $\rho$  can be distilled. The condition (II.2) is easy to check and we will use it in our discussion of dynamical aspects of distillability.

### B. Peres-Horodecki criterion and bound entanglement

To detect entangled states of two qutrits, we apply Peres-Horodecki criterion of separability [10, 11]. From this criterion follows that any state  $\rho$  for which its partial transposition  $\rho^{\text{PT}}$  is non - positive (NPPT state), is entangled. One defines also *negativity* of the state  $\rho$  as

$$N(\rho) = \frac{\|\rho^{\text{PT}}\|_{\text{tr}} - 1}{2} \quad (\text{II.3})$$

$N(\rho)$  is equal to the absolute value of the sum of the negative eigenvalues of  $\rho^{\text{PT}}$  and is an entanglement monotone [24], however it cannot detect entangled states which are positive

under partial transposition (PPT states). Such states exist [25] and as was shown in Ref. [12], are not distillable. They are called *bound entangled PPT states*. Up to now, it is not known if there exist bound entangled NPPT states [26].

To detect some of bound entangled PPT states we can use the realignment criterion of separability [27, 28]. The criterion states that for any separable state  $\rho$  of a compound system, the matrix  $R(\rho)$  with elements

$$\langle m | \otimes \langle \mu | R(\rho) | n \rangle \otimes | \nu \rangle = \langle m | \otimes \langle n | \rho | \mu \rangle \otimes | \nu \rangle \quad (\text{II.4})$$

has a trace norm not greater than 1. So if the *realignment negativity* defined by

$$N_R(\rho) = \max \left( 0, \frac{\|R(\rho)\|_{\text{tr}} - 1}{2} \right) \quad (\text{II.5})$$

is greater than zero, the state  $\rho$  is entangled. In the case of two qubits, the measure (II.5) cannot detect all NPPT states [29], but for larger dimension the criterion is capable of detecting some bound entangled PPT states [27].

## III. TIME EVOLUTION OF THREE-LEVEL ATOMS

To study the dynamics of entanglement between three-level atoms we consider the model introduced by Agarwal and Patnaik [7]. We start with the short description of the model. Consider two identical three - level atoms ( $A$  and  $B$ ) in the  $V$  configuration. The atoms have two near - degenerate excited states  $|1_\alpha\rangle$ ,  $|2_\alpha\rangle$  ( $\alpha = A, B$ ) and ground states  $|3_\alpha\rangle$ . Assume that the atoms interact with the common vacuum and that transition dipole moments of atom  $A$  are parallel to the transition dipole moments of atom  $B$ . Due to this interaction, the process of spontaneous emission from two excited levels to the ground state take place in each individual atom but a direct transition between excited levels is not possible. Moreover, the coupling between two atoms can be produced by the exchange of the photons, but in such atomic system there is also possible the radiative process in which atom  $A$  in the excited state  $|1_A\rangle$  loses its excitation which in turn excites atom  $B$  to the state  $|2_B\rangle$ . This effect manifests by the cross coupling between radiation transitions with orthogonal dipole moments. The evolution of this atomic system can be described by the following master equation [7]

$$\frac{d\rho}{dt} = i[H, \rho] + (L^A + L^B + L^{AB})\rho \quad (\text{III.1})$$

where

$$H = \sum_{k=1}^2 \Omega_{k3} (\sigma_{k3}^A \sigma_{3k}^B + \sigma_{k3}^B \sigma_{3k}^A) + \sum_{\alpha=A,B} \Omega_{\alpha 3} (\sigma_{23}^\alpha \sigma_{31}^{-\alpha} + \sigma_{32}^\alpha \sigma_{13}^{-\alpha}) \quad (\text{III.2})$$

and for  $\alpha = A, B$

$$L^\alpha \rho = \sum_{k=1}^2 \gamma_{k3} (2\sigma_{3k}^\alpha \rho \sigma_{k3}^\alpha - \sigma_{a3}^\alpha \sigma_{3k}^\alpha \rho - \rho \sigma_{k3}^\alpha \sigma_{3k}^\alpha) \quad (\text{III.3})$$



where

$$a = 1 - 2x - y, \quad b = 1 - x - 2y, \quad c = x + y$$

and the matrix on the right hand side of (IV.2) has two negative leading principal minors (other minors are positive), so

$$\text{tr}_B \rho_{\text{as}} \otimes \mathbb{1} - \rho_{\text{as}} \not\geq 0$$

Similarly

$$\mathbb{1} \otimes \text{tr}_A \rho_{\text{as}} - \rho_{\text{as}} \not\geq 0$$

The interesting examples of nontrivial asymptotic states are given by the separable initial states where the one atom is in the excited state and the other is in the ground state or two atoms are in different excited states. In all such cases, the created entanglement is free and can be distilled.

Now we consider the possibility of creating free stationary entanglement from the bound initial entanglement. As the initial states we take the family [30]

$$\rho_\alpha = \frac{2}{7} |\Psi_0\rangle\langle\Psi_0| + \frac{\alpha}{7} P_+ + \frac{5-\alpha}{7} P_-, \quad 3 < \alpha \leq 4 \quad (\text{IV.3})$$

where

$$|\Psi_0\rangle = \frac{1}{\sqrt{3}} \sum_{j=1}^3 |j_A\rangle \otimes |j_B\rangle,$$

$$P_+ = \frac{1}{3} (P_{|1_A\rangle \otimes |2_B\rangle} + P_{|2_A\rangle \otimes |3_B\rangle} + P_{|3_A\rangle \otimes |1_B\rangle})$$

and

$$P_- = \frac{1}{3} (P_{|2_A\rangle \otimes |1_B\rangle} + P_{|3_A\rangle \otimes |2_B\rangle} + P_{|1_A\rangle \otimes |3_B\rangle}).$$

The states (IV.3) are constructed as follows: we prepare the maximally entangled state  $|\Psi_0\rangle$  and add some specific noise resulting in the mixing of  $|\Psi_0\rangle$  with separable states  $P_+$  and  $P_-$ . For a special choice of mixing parameter such prepared states have positive partial transposition but are entangled, as can be shown by computing the realignment negativity. For  $\rho_\alpha$  it is given by

$$N_R(\rho_\alpha) = \frac{1}{21} \left( \sqrt{3\alpha^2 - 15\alpha + 19} - 1 \right) \quad (\text{IV.4})$$

and is obviously positive for  $3 < \alpha \leq 4$ . So the states (IV.3) are bound entangled and the entanglement initially present in  $|\Psi_0\rangle$  cannot be extracted from them, for any number of copies of the states. It is worth to notice that recently the bound entanglement was created experimentally in the system of three qubits [31].

Although the states (IV.3) are not diagonal, one can check that the corresponding asymptotic states have the same form as in the diagonal case. In fact, for all initial states  $\rho_\alpha$  there is only one asymptotic state  $\rho_{\text{as}}$  given by

$$x = y = \frac{5}{56} \quad \text{and} \quad t = \frac{9}{14}$$

By the above discussion, this state is entangled and moreover its entanglement is distillable. Thus we have shown that the physical process of spontaneous emission in the radiatively coupled three - level atoms can transform initial bound entanglement into free distillable entanglement of the asymptotic state.

## V. DELAYED CREATION OF DISTILLABLE ENTANGLEMENT

In this section we study in details the evolution of entanglement of the bound entangled initial states (IV.3) for  $3 < \alpha \leq 4$ , beyond the strong correlation regime. In that case, the asymptotic state is trivial, but some transient entanglement between the atoms can be produced. For simplicity, we consider such atomic configuration for which the cross coupling coefficients are equal to zero. One can check that the initial states (IV.3) will evolve into the states of the form

$$\rho_\alpha(t) = \begin{pmatrix} \rho_{11} & 0 & 0 & 0 & \rho_{15} & 0 & 0 & 0 & \rho_{19} \\ 0 & \rho_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_{33} & 0 & 0 & 0 & \rho_{37} & 0 & 0 \\ 0 & 0 & 0 & \rho_{44} & 0 & 0 & 0 & 0 & 0 \\ \rho_{51} & 0 & 0 & 0 & \rho_{55} & 0 & 0 & 0 & \rho_{59} \\ 0 & 0 & 0 & 0 & 0 & \rho_{66} & 0 & \rho_{68} & 0 \\ 0 & 0 & \rho_{73} & 0 & 0 & 0 & \rho_{77} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_{86} & 0 & \rho_{88} & 0 \\ \rho_{91} & 0 & 0 & 0 & \rho_{95} & 0 & 0 & 0 & \rho_{99} \end{pmatrix} \quad (\text{V.1})$$

where all non-zero matrix elements are time dependent.

Numerical analysis indicates that during the time evolution the realignment negativity (II.5) of the initial state very rapidly goes to zero, so the system almost immediately disentangle. To consider possible creation of free entanglement, let us first check if PPT condition can be violated during such evolution. After taking the partial transposition, the state (V.1) becomes

$$\rho_\alpha(t)^{\text{PT}} = \begin{pmatrix} \rho_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{37} \\ 0 & \rho_{22} & 0 & \rho_{15} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_{33} & 0 & 0 & 0 & \rho_{19} & 0 & 0 \\ 0 & \rho_{51} & 0 & \rho_{44} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho_{55} & 0 & 0 & 0 & \rho_{68} \\ 0 & 0 & 0 & 0 & 0 & \rho_{66} & 0 & \rho_{59} & 0 \\ 0 & 0 & \rho_{91} & 0 & 0 & 0 & \rho_{77} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_{95} & 0 & \rho_{88} & 0 \\ \rho_{37} & 0 & 0 & 0 & \rho_{86} & 0 & 0 & 0 & \rho_{99} \end{pmatrix} \quad (\text{V.2})$$

One can check that determinant  $d$  of the matrix (V.2) equals

$$d = (\rho_{22}\rho_{44} - |\rho_{15}|^2)(\rho_{33}\rho_{77} - |\rho_{19}|^2)(\rho_{66}\rho_{88} - |\rho_{59}|^2) \\ \times (\rho_{11}\rho_{55}\rho_{99} - \rho_{55}|\rho_{37}|^2 - \rho_{11}|\rho_{86}|^2) \quad (\text{V.3})$$

We can show numerically that (V.3) changes the sign, since the last factor is positive for all  $t < t_N$  and becomes negative if  $t > t_N$ , for some  $t_N > 0$ , and the remaining factors are positive. Moreover, all other leading principal minors of the matrix

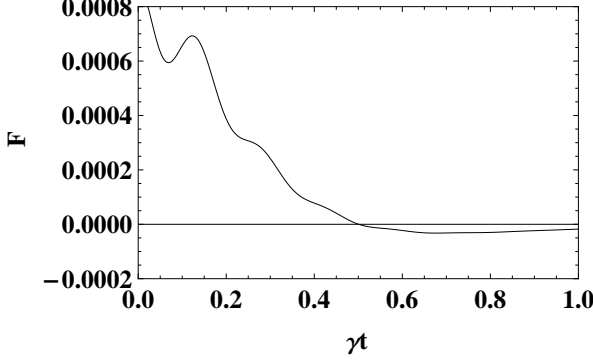


FIG. 1: The time evolution of the factor  $F = \rho_{11}\rho_{55}\rho_{99} - \rho_{55}|\rho_{37}|^2 - \rho_{11}|\rho_{86}|^2$  in (V.3) for the initial state (IV.3) with  $\alpha = 3.6$  and the interatomic distance  $R = 0.2\lambda$ .

(V.2) are always positive. So the evolution of the bound entangled state (IV.3) has the interesting property: for all  $t < t_N$  the states  $\rho_\alpha(t)$  are PPT and then suddenly they become NPPT states (see FIG. 1).

Now we discuss distillability of the states  $\rho_\alpha(t)$ . Since we cannot exclude the possibility that there are NPPT states which are non-distillable, we try to apply the reduction criterion of entanglement. As we know from the discussion in Sect. II, any state violating this criterion is necessarily distillable. By direct computations we show that the matrix

$$\text{tr}_B \rho_\alpha(t) \otimes \mathbb{1} - \rho_\alpha(t)$$

equals to

$$\begin{pmatrix} r_{11} & 0 & 0 & 0 & -\rho_{15} & 0 & 0 & 0 & -\rho_{19} \\ 0 & r_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & r_{33} & 0 & 0 & 0 & -\rho_{37} & 0 & 0 \\ 0 & 0 & 0 & r_{44} & 0 & 0 & 0 & 0 & 0 \\ -\rho_{15} & 0 & 0 & 0 & r_{55} & 0 & 0 & 0 & -\rho_{59} \\ 0 & 0 & 0 & 0 & 0 & r_{66} & 0 & -\rho_{68} & 0 \\ 0 & 0 & -\rho_{73} & 0 & 0 & 0 & r_{77} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\rho_{86} & 0 & r_{88} & 0 \\ -\rho_{91} & 0 & 0 & 0 & -\rho_{95} & 0 & 0 & 0 & r_{99} \end{pmatrix} \quad (\text{V.4})$$

where

$$r_{kk} = \begin{cases} \rho_{11} + \rho_{22} + \rho_{33} - \rho_{kk}, & k = 1, 2, 3 \\ \rho_{44} + \rho_{55} + \rho_{66} - \rho_{kk}, & k = 4, 5, 6 \\ \rho_{77} + \rho_{88} + \rho_{99} - \rho_{kk}, & k = 7, 8, 9 \end{cases} \quad (\text{V.5})$$

We compute that leading principal minors of the matrix (V.4) which can change the sign during the evolution, and find the following expressions

$$\begin{aligned} m_5 &= r_{22}r_{33}r_{44} (r_{11}r_{55} - |\rho_{15}|^2) \\ m_6 &= r_{22}r_{33}r_{44}r_{66} (r_{11}r_{55} - |\rho_{15}|^2) \\ m_7 &= r_{22}r_{44}r_{66} (r_{11}r_{55} - |\rho_{15}|^2) (r_{33}r_{77} - |\rho_{37}|^2) \\ m_8 &= r_{22}r_{44} (r_{11}r_{55} - |\rho_{15}|^2) (r_{33}r_{77} - |\rho_{37}|^2) (r_{66}r_{88} - |\rho_{68}|^2) \end{aligned} \quad (\text{V.6})$$

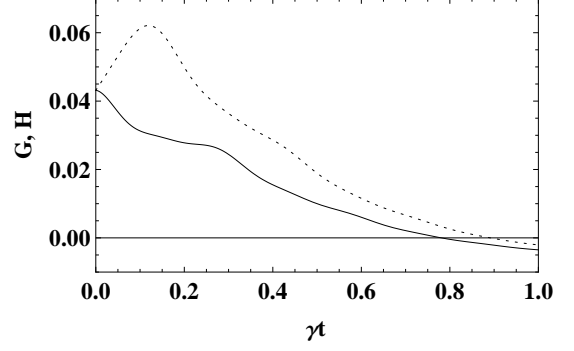


FIG. 2: The time evolution of the factors  $G = r_{33}r_{77} - |\rho_{37}|^2$  (dotted line) and  $H = r_{66}r_{88} - |\rho_{68}|^2$  (solid line) in (V.6) for the initial state (IV.3) with  $\alpha = 3.6$  and  $R = 0.2\lambda$ .

where  $m_k$  for  $k = 5, 6, 7, 8$  are determinants of principal  $k \times k$  submatrices of the matrix (V.4). It turns out that the factor  $r_{11}r_{55} - |\rho_{15}|^2$  is always positive, but as follows from the numerical analysis,  $r_{33}r_{77} - |\rho_{37}|^2$  as well as  $r_{66}r_{88} - |\rho_{68}|^2$  change the sign during the evolution (see FIG. 2). Let  $t_D$  be the time at which the factor  $r_{66}r_{88} - |\rho_{68}|^2$  changes the sign. We see that  $t_D > 0$ , so only after that time the matrix (V.4) becomes non-positive. It means that for  $t > t_D$ , the states  $\rho_\alpha(t)$  are necessarily distillable. One can check that  $t_D > t_N$ , and we see that the initial bound entangled state (IV.3) evolves in the remarkable way: for all  $t \leq t_N$  it is PPT, for  $t_N < t \leq t_D$  it is NPPT but a priori can be non-distillable and only after  $t_D$  it becomes distillable. To show this in the explicit way, let us introduce the measure of the violation of reduction criterion, defined as

$$N_{\text{red}}(\rho) = \max \left( 0, -\lambda_{\min}^{\text{red}} \right) \quad (\text{V.7})$$

where  $\lambda_{\min}^{\text{red}}$  is the minimal eigenvalues of the matrix

$$\rho_{\text{red}} = \text{tr}_B \rho \otimes \mathbb{1} - \rho$$

The quantity (V.7) can be called the reduction negativity of the state  $\rho$ . For the bound entangled initial state (IV.3) the evolution of negativity and reduction negativity is given below (FIG. 3). So we observe in the system the phenomenon of delayed sudden birth of distillable entanglement. The physical reason for the appearance of this phenomenon can be explained as follows. During the time evolution of the system, the process of the photon exchange produces coherence between the states  $|1_A\rangle \otimes |3_B\rangle$  and  $|3_A\rangle \otimes |1_B\rangle$ , so the value of  $|\rho_{37}|$  starts to grow. Similarly, the same process causes the production of coherence between states  $|2_A\rangle \otimes |3_B\rangle$  and  $|3_A\rangle \otimes |2_B\rangle$ , so  $|\rho_{68}|$  also grows. Notice that non-zero value of  $|\rho_{37}|$  or  $|\rho_{68}|$  is necessary for the possibility of creation of distillable entanglement. But this condition is not sufficient since the populations of states of two-atomic system also evolve in time. We see from the formula (V.6) that there is a threshold for the reduction negativity (V.7) at which the system becomes distillable.

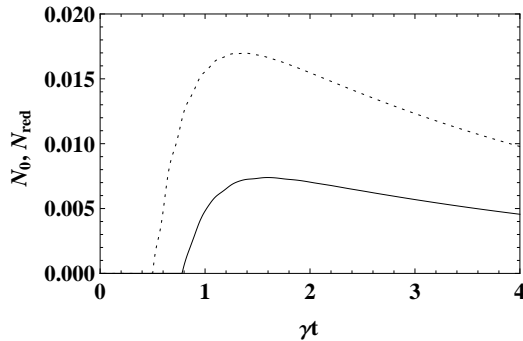


FIG. 3: The time evolution negativity  $N_0$  (dotted line) and reduction negativity  $N_{\text{red}}$  (solid line) for the initial state (IV.3) with  $\alpha = 3.6$  and  $R = 0.2\lambda$ .

The numerical value of  $t_D$  depends on the choice of the parameter  $\alpha$  and the interatomic distance  $R$ . For the initial state with  $\alpha = 3.6$  and the distance  $R = 0.2\lambda$ , we obtain  $t_D\gamma \approx 0.78$  whereas  $t_N\gamma \approx 0.49$ .

## VI. CONCLUSIONS

We have studied the dynamics of entanglement in the system of three-level atoms in the  $V$  configuration, coupled to

the common vacuum. In the case of small (compared to the radiation wavelength) separation between the atoms, the system has nontrivial asymptotic states which can be entangled even if the initial states were separable. For the large class of separable initial states the asymptotic states are not only entangled but also distillable. The same is true for some class of bound entangled initial states. Thus we have shown that the dynamics of the system can transform bound entanglement into the free distillable entanglement of stationary states. For the atoms separated by larger distances only some transient entanglement can exist but still the dynamical generation of entanglement is possible. We have shown that this happens also for the class of bound entangled initial states. Moreover we have demonstrated that such states evolve in a very peculiar way: they almost immediately disentangle after the atoms begin to interact with the vacuum, then for some finite period of time there is no entanglement and suddenly at some time the entanglement starts to build up. But this entanglement a priori can be nondistillable. We have analysed this problem using the reduction criterion of separability and found that the free entanglement surely appears in the system after some additional period of time.

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